Fugitive Methane Detection and Localization With Small Unmanned Aerial Systems: Challenges and Opportunities

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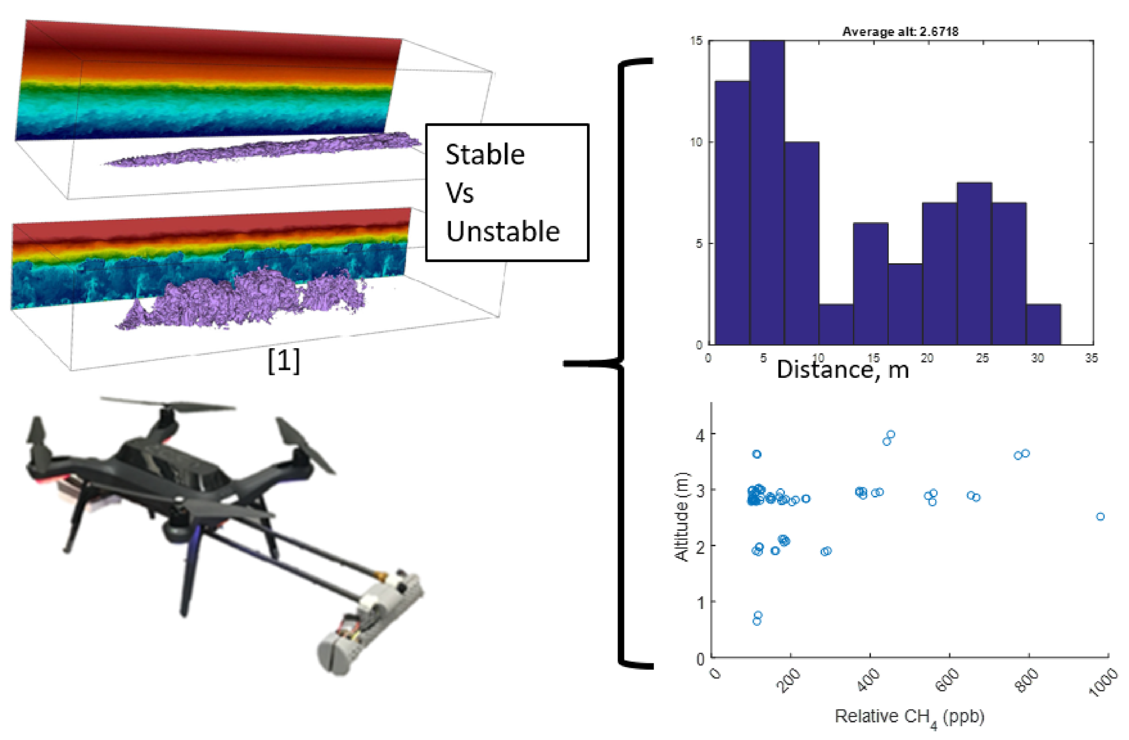
Introduction

Natural gas is one of our main methods to generate power today. Utility companies that provide this gas are tasked with maintaining and surveying leaks. These leaks are referred to as fugitive methane emissions and detecting these fugitive gases can be pivotal to preventing incidents such as the San Bruno explosion, killing 8 and injuring dozens due to a gas leak going undetected. Recently, using NASA technology onboard low cost vertical takeoff and landing (VTOL) small unmanned aerial systems (sUAS) we can detect fugitive methane at 1 ppb (parts per billion) levels.

Challenges in Detection

General challenges include: FAA regulations (no flights over people), battery life, and complex dynamic plume behavior. Factors that impact detection can be: propeller wash, sensor placement, wind, and mechanical/electrical noises. Even distance to source and flight altitudes can change the probability of detection (Sigmoid like) scaling with topology and atmospheric stability. Localization by CFD approaches are costly making real-time estimations and visualizations difficult.



Quasi-Steady Inversion

Following the work by Matthes et al (2005), Carslaw (1959), and Roberts (1923) the solution to a single point source advection diffusion equation (ADE) can be solved for a dynamic system approximately by making a quasi-steady state assumption if the variance and transient behavior of the wind small. W_0 is the Lambert function.

$$\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x_i^2} + v\frac{\partial C}{\partial x_i} = 2q_0\delta(t-t_0)\delta(x_i - x_{i0})$$

$$\bar{C}(\bar{x}_i, x_0, q_o)_i = \frac{q_0 \exp(\frac{\bar{v}(\bar{x}_i - x_0)}{2D})}{\pi^{\frac{2}{3}} Dd}$$

$$d_i(C_i, x_0, q_0) \approx \frac{2D}{\bar{v}} W_0\left(\frac{\bar{v} q_0}{4\pi D^2 C_i}\exp\left(\frac{v}{2D}(\bar{x}_i - x_0)\right)\right)$$

$$\min_{q_0, x_0} : \sum_{i,j=1}^{m} (y_{0,i}(x_0, q_0) - y_{0,j}(x_0, q_0))^2$$

Adaptive Search Model

In the foraging literature the Levy walk has been shown to be effective at searching sparse environments. However, Brownian motion is more efficient in dense areas. This adaptive search model [5] can switch dynamically from Levy to Brownian based on finding targets using tumble probability $P(x(t))$, $x(t)$ is governed by the stochastic differential equation (SDE) below

$$P(x(t)) = e^{-x(t)}, \quad 0 \le x \le 5$$

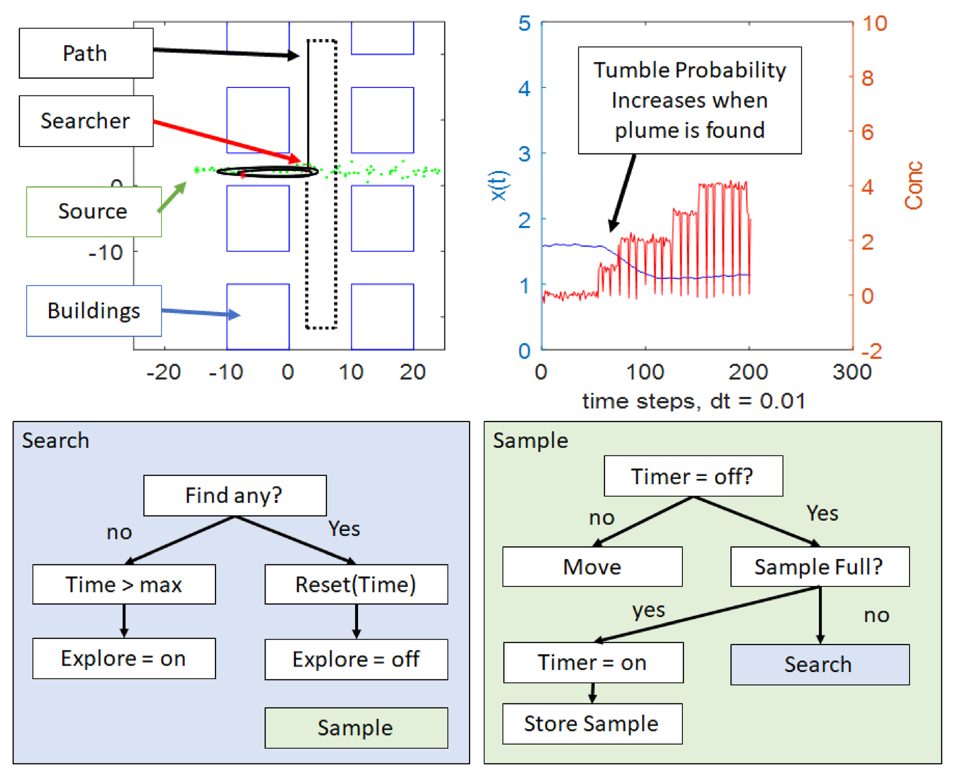
$$\dot{x} = -\frac{\partial U}{\partial x}A + \epsilon, \begin{cases} U = (x-h)^2, \epsilon : \begin{cases} H = \frac{1}{2}, \mathrm{N}(0,\sigma) \\ H \neq \frac{1}{2}, \mathrm{fGn} \end{cases} \\ A = \max(A_{min}, \alpha(t)) \end{cases}$$

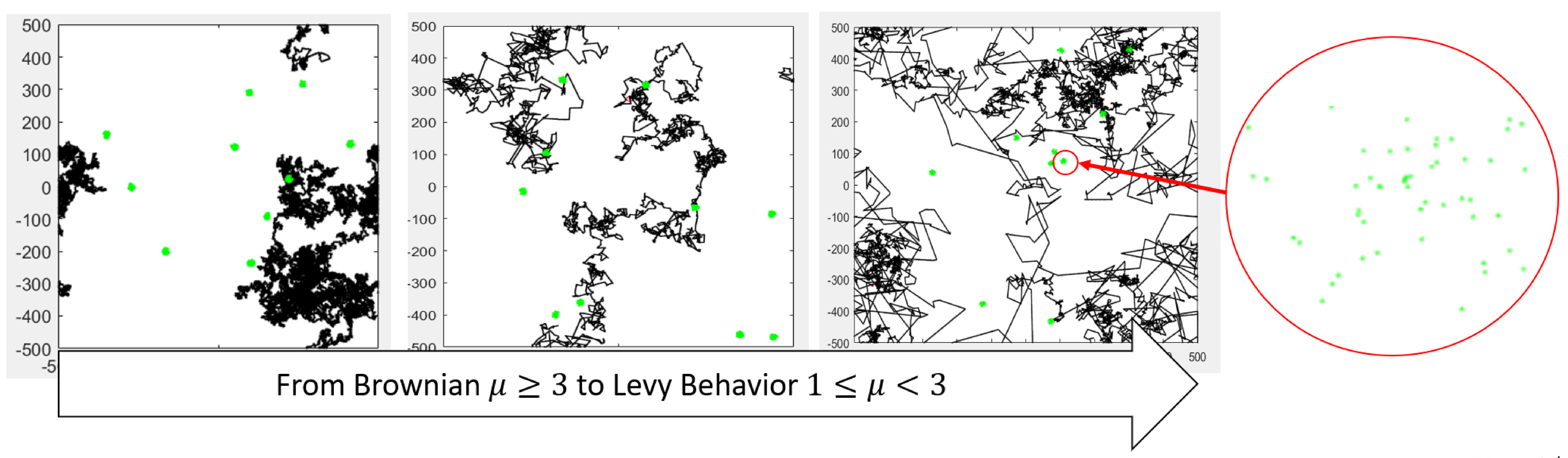
$$\alpha_k = C_\alpha \alpha_{k-1} + k_t F \begin{cases} F = 1, \text{found target} \\ F = 0, \text{otherwise.} \end{cases}$$

we extend [5] by adding, fGn, defined as $Y_j = B_H(j+1) - B_H(j)$ and fraction Brownian motion is given below.

Adaptive Search and Localization

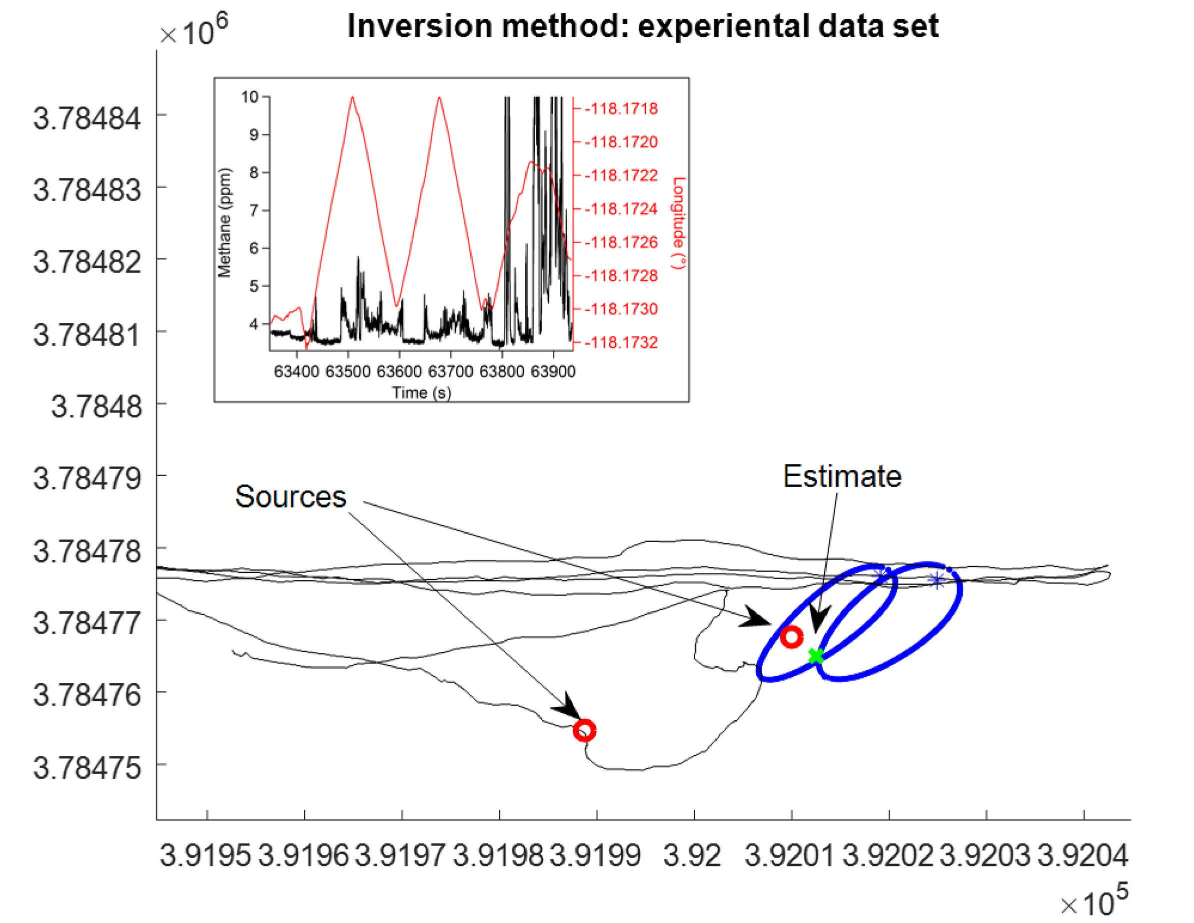
The adaptive search model has shown to adjust from Brownian motion to Levy walks in a 2D random search. By reducing the problem to a 1D path problem (i.e. survey route) adding decision trees and modeling fugitive gas with a small time scale filament model [4] we have the opportunity to optimize random search for application. Gather enough information to form a sample(s) to use in the inversion method for a Zeroth order approximation of source localization (x_0,y_0) and quantification (q_0).





$$B_H(x) \approx \sum \phi(x-y)B(\Delta y) \qquad \phi(x) = \frac{\Gamma(H+1-d/2+||x||)}{\Gamma(||x||+1)\Gamma(H+1-d/2)} \approx \frac{||x||^{H-d/2}}{\Gamma(H+1-d/2)}$$

Experimental Results



Using the quasi-steady inversion method on experimental data we can see the results from just two samples (blue) in the presence of two sources (red). Only taking a small section of raw data from each longitudinal pass we can approximate the source (green) from our measurement with the OPLS [3].

Future Research

This work hopes to optimize this adaptive search strategy efficiency $\eta = N/L$ (N is the number of targets found and L is the total distance traveled) through transition parameters (C_α, A_{min}, and k_t) the potential (h), and the choice of noise (i.e. Gaussian or fGn) by means of evolutionary algorithms. Furthermore, we want to answer how the level of noise σ and how the Hurst parameter H, stochastically shift the tumble probability through $x(t)$. Once we have an optimal model we look to compare with current methods (i.e. Zig-Zag, spiral surge [2]), and other gradient or flux based approaches (stochastic gradient descent, fluxotaxis, infotaxis etc.).

References

[1] Matheou et al. *Environ Fluid Mech.*, 2016.
[2] Li et al. *Int. Conf. on Rob. and Biomim.*, 2009.
[3] Smith et al. *ICUAS Miami.*, 2017.
[4] Farrell et al. *Env. Fluid Mech.*, 2002.
[5] Nurzaman et al. *PLos ONE*, 2011.

Very Slow Diffusion Processes and its Regional Analysis

ICERM FPDE Workshop 2018, Brown University

Presenter: Dr. YangQuan Chen

The MESA Lab, University of California, Merced
Joint work with Ruiyang Cai (Donghua University, China) and Yuquan Chen (University of Science and Technology of China)

June 21, 2018

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Background

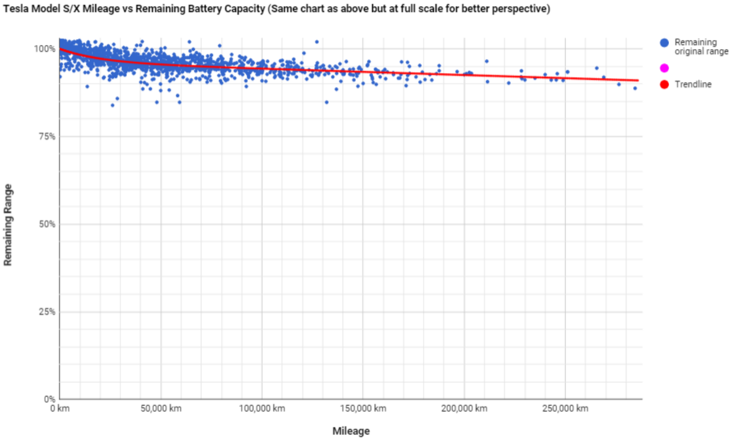


Figure: Tesla Model S/X Mileage VS Remaining Battery Capacity

Background

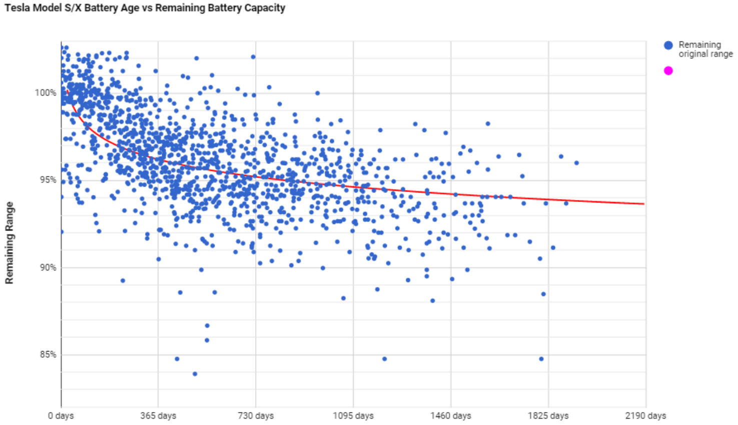


Figure: Tesla Model S/X Battery Age VS Remaining Battery Capacity

Some Links

- https://docs.google.com/spreadsheets/d/t024bMoRiDPIDialGnuKPsg/edit#gid=1669966328
- https://docs.google.com/spreadsheets/d/t024bMoRiDPIDialGnuKPsg/edit#gid=154312675

For more details, see [1].

Background

Figure: Human feet as a geological force

- https://twitter.com/PaulMMCooper/status/1007612133356572672

Different tails to describe decay rate

$$\textit{Fast} \quad \frac{\exp(-\lambda t)}{t^{\alpha}} \rightarrow \frac{\exp(-\lambda t^{\beta})}{t^{\alpha}} \rightarrow \frac{1}{t^{\alpha}} \rightarrow \frac{1}{(\log t)^{\alpha}} \quad \textit{Slow}$$

$$\textit{ultra-fast} \rightarrow \textit{very-fast} \rightarrow \textit{power-law} \rightarrow \textit{ultra-slow}$$

For the above data, $t^{-\alpha}$ is too fast while $(\log t)^{-\alpha}$ is too slow

Very Slow between power-law and ultra-slow?

Yes!

A new tail for very-slow: $\dfrac{\log t}{t^{\alpha}}$

Image of these kernels: x label-t

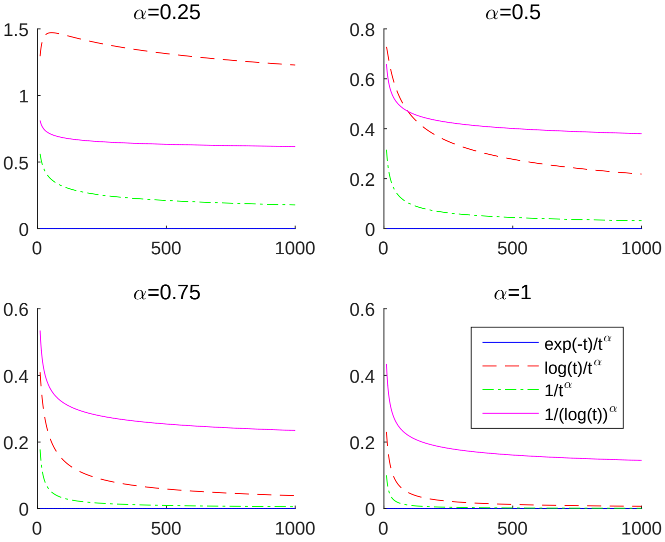
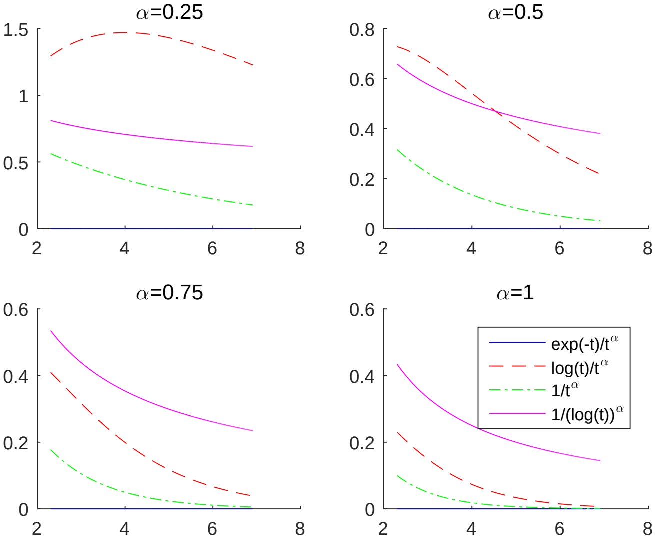
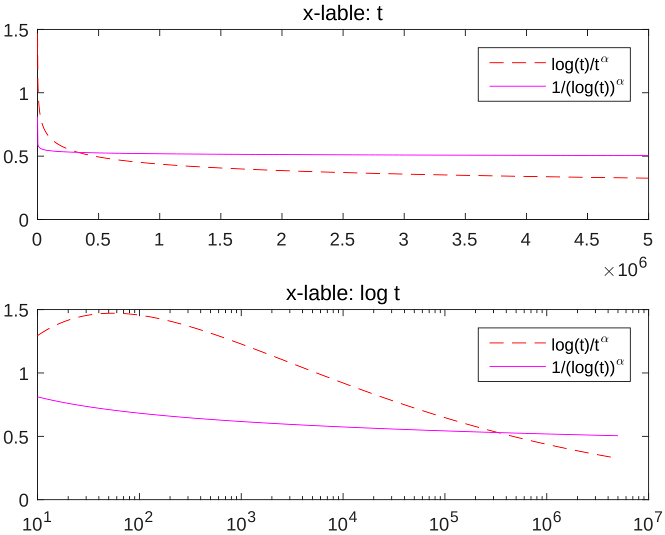


Image of these kernels: x label-log t



Comparison between $\frac{\log t}{t^{\alpha}}$ and $\frac{1}{(\log t)^{\alpha}}$ when $\alpha = 0.25$



Laplace transform of the very slow kernel

The Laplace transform of the very slow kernel is

$$\mathfrak{L}\left(\frac{\log t}{t^{\alpha}}\right)(s) = s^{\alpha-1}\Gamma(1-\alpha)\left(\psi(1-\alpha) - \log s\right),$$

where $\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ denotes the digamma function.

Realization of the kernel in engineering by Prony

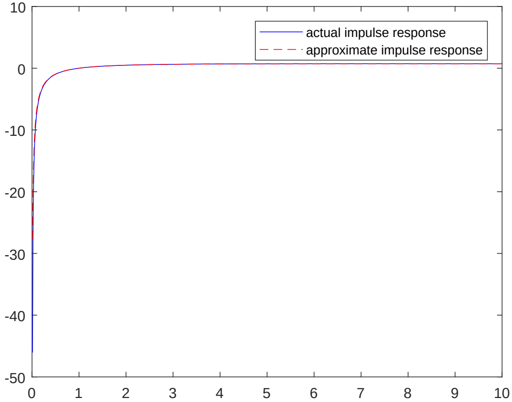


Figure: Prony: α=0.5, order=12, t=10, Ts=0.01

Realization of the kernel in engineering by Prony

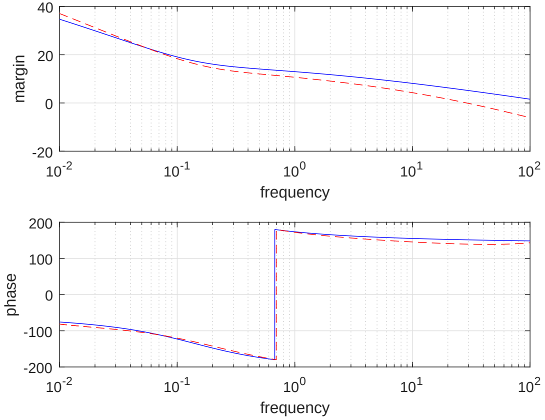


Figure: Bode diagram: α=0.5, order=12

Realization of the kernel in engineering by Prony

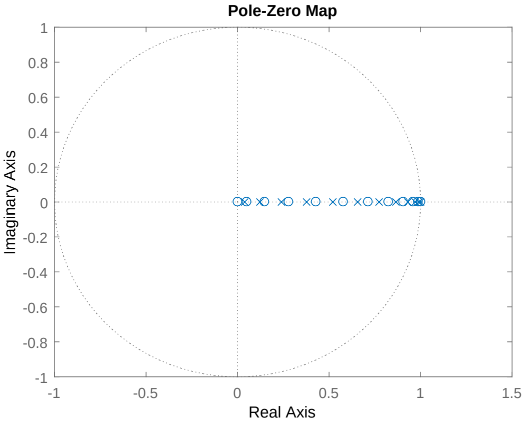


Figure: Pole-Zero: α=0.5, order=12

Definitions

- α-th order fractional integral:

$$ {}_0I_t^{\alpha} f(t) \triangleq \frac{1}{\Gamma(\alpha)} \int_0^t \frac{\log(t-s)}{(t-s)^{1-\alpha}} f(s) ds $$

- α-th order Riemann-Liouville type fractional derivative:

$$ {}_0D_t^{\alpha} f(t) \triangleq \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{\log(t-s)}{(t-s)^{\alpha}} f(s) ds $$

- α-th order Caputo type fractional derivative:

$$ {}_0^C D_t^{\alpha} f(t) \triangleq \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\log(t-s)}{(t-s)^{\alpha}} f'(s) ds $$

Properties

- $D^{\alpha} f(t) = \frac{d}{dt} I^{1-\alpha} f(t)$
- ${}_0I_t^{\beta}\, {}_0I_t^{\alpha} f(t) = {}_0I_t^{\alpha}\, {}_0I_t^{\beta} f(t) \neq {}_0I_t^{\alpha+\beta} f(t)$
- ${}_0D_t^{\beta}\, {}_0D_t^{\alpha} f(t) = {}_0D_t^{\alpha}\, {}_0D_t^{\beta} f(t) \neq {}_0D_t^{\alpha+\beta} f(t)$
- ${}_0^CD_t^{\beta}\, {}_0^CD_t^{\alpha} f(t) = {}_0^CD_t^{\alpha}\, {}_0^CD_t^{\beta} f(t) \neq {}_0^CD_t^{\alpha+\beta} f(t)$

Integral and derivative of some special functions

$f(t) = 1$:

- ${}_0I_t^\alpha 1 = \frac{t^\alpha}{\Gamma(\alpha+1)}\left(\log t - \frac{1}{\alpha}\right)$
- ${}_0D_t^\alpha 1 = \frac{1}{\Gamma(1-\alpha)} t^{-\alpha} \log t$
- ${}_0^C D_t^\alpha 1 = 0$

Integral and derivative of some special functions

$f(t) = t^k$:

- ${}_0I_t^{\alpha} t^k = \frac{\Gamma(k+1)}{\Gamma(k+\alpha+1)} t^{k+\alpha} \left(\log t + \psi(\alpha) - \psi(k+\alpha+1)\right)$
- $${}_0D_t^{\alpha} t^k = \left(\frac{\Gamma(k+1)}{\Gamma(k-\alpha+2)} + \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} \left(\psi(1-\alpha) - \psi(k-\alpha+2)\right)\right) t^{k-\alpha} + \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} t^{k-\alpha} \log t$$
- ${}_0^C D_t^{\alpha} t^k = \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} t^{k-\alpha} \left(\log t + \psi(1-\alpha) - \psi(k-\alpha+1)\right)$

Integral and derivative of some special functions

$f(t) = \exp(t)$:

- ${}_0I_t^{\alpha} \exp(t) = \frac{\psi(\alpha)(\Gamma(\alpha)-\Gamma(\alpha,t))}{\Gamma(\alpha)} \exp(t) - \mathfrak{L}^{-1}\left(\frac{\log s}{s^{\alpha}(s-1)}\right)(t)$
- ${}_0D_t^{\alpha} \exp(t) = {}_0^C D_t^{\alpha} \exp(t) = \frac{d}{dt}\left({}_0I_t^{1-\alpha} \exp(t)\right)$

where $\Gamma(\alpha, t)$ is the upper incomplete Gamma function defined by

$$\Gamma(\alpha, t) = \int_t^{\infty} x^{\alpha-1} \exp(-x) dx$$

Realization of $\mathfrak{L}^{-1}\left(\frac{\log s}{s^\alpha(s-1)}\right)(t)$ by NILT and Prony

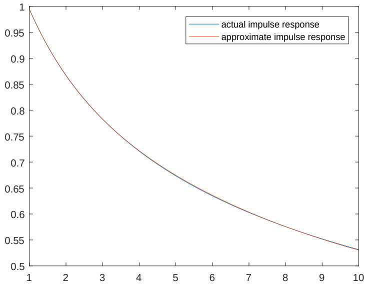


Figure: NILT: α=0.4, order=3, t=10, Ts=0.001

$$G(z^{-1}) = \frac{0.9942z^2 - 1.988z + 0.9936}{z^3 - 2.999z^2 + 2.998z - 0.9992}$$

Realization of $\mathfrak{L}^{-1}\left(\frac{\log s}{s^{\alpha}(s-1)}\right)(t)$ by NILT and Prony

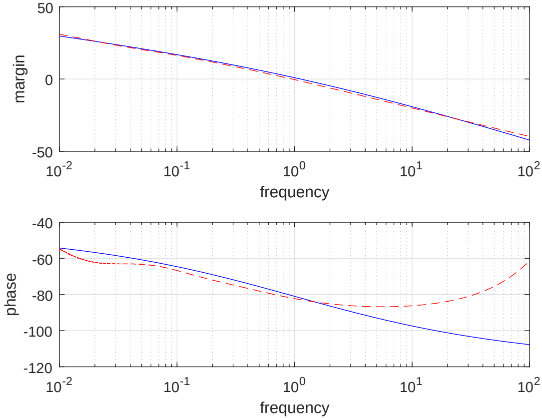


Figure: Bode diagram: α=0.4, order=3, Ss=0.01

Realization of $\mathfrak{L}^{-1}\left(\frac{\log s}{s^{\alpha}(s-1)}\right)(t)$ by NILT and Prony

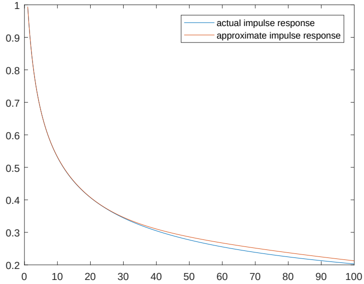


Figure: NILT: α=0.4, order=5, t=100, Ts=0.01

$$G(z^{-1}) = \frac{0.9942z^4 - 3.904z^3 + 5.749z^2 - 3.761z + 0.9225}{z^5 - 4.925z^4 + 9.703z^3 - 9.555z^2 + 4.705z - 0.9264}$$

Realization of $\mathfrak{L}^{-1}\left(\frac{\log s}{s^{\alpha}(s-1)}\right)(t)$ by NILT and Prony

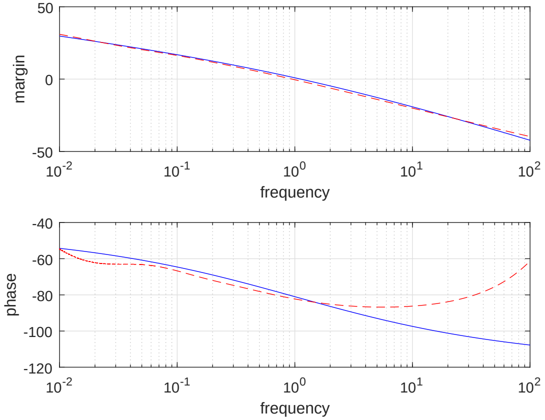


Figure: Bode diagram: α=0.4, order=5, Ss=0.01

- MATLAB NILT toolbox: https://www.mathworks.com/matlabcentral/fileexchange/39035-numerical-inverse-laplace-transform?s_tid=srchtitle
- More theory and applications of NILT, see [2]
- MATLAB Prony toolbox: see [3]

What is regional analysis

In 1988, El Jai et al. first introduced the "regional analysis" [4].

Briefly speaking, regional analysis is to control, observe, stabilize or detect the considered systems on a sub-region of the whole domain of interest.

Our works on the regional analysis of fractional order PDEs

Sub-diffusion

- Controllability (normal, gradient, boundary; exact, approximate)
- Observability (normal, gradient, boundary; exact, approximate)
- Detection of unknown sources
- Spreadability
- Stability and stabilizability (normal, boundary)

Ultra-slow diffusion

- Controllability (normal, gradient; exact, approximate)
- Observability (normal, gradient; exact, approximate)

Main works

- Ge F, Chen YQ, Kou C, Podlubny I.Fractional Calculus and Applied Analysis, 2016.
- Ge F, Chen YQ, Kou C. Journal of Mathematical Analysis and Applications, 2016.
- Ge F, Chen YQ, Kou C. Automatica, 2016.
- Ge F, Chen YQ, Kou C. Journal of Mathematical Analysis and Applications, 2016.
- Ge F, Chen YQ, Kou C. Automatica, 2017.
- Ge F, Chen YQ, Kou C. IMA Journal of Mathematical Control and Information, 2017.
- Ge F, Chen YQ, Kou C. Springer, 2018 [5]. The first monograph about the regional analysis of fractional diffusion processes.

Motivation

Figure: Monitoring the fire and managing to put it out

How many sensors (UAVs) and actuators (fire extinguishers)?

Why we need regional analysis

- More efficient
- Reduction in the number of actuators and sensors
- Reduce the computational requirements
- Discuss the systems which are not controllable/observable/stable/detectable on the whole domain

Problem statement

Consider the following time fractional order diffusion system with the Caputo type of our new time fractional derivative:

$$\begin{cases} {}_a^C D_t^{\alpha} y(x,t) = Ay(x,t) + Bu(t) \text{ in } \Upsilon, \\ y(x,a) = y_0(x) \text{ in } \Omega, \\ y(\xi,t) = 0 \text{ on } \Sigma, \end{cases} \tag{1}$$

where

- $\Upsilon = \Omega \times [a,b]$, $\Sigma = \partial\Omega \times [a,b]$ and $0 < \alpha < 1$.
- A is an infinitesimal generator of a C_0-semigroup $\{T(t)\}$ on the Hilbert space $L^2(\Omega)$, while $-A$ a uniformly elliptic operator.
- The initial vector $y_0 \in L^2(\Omega)$ is unknown in observability problem.
- $u(t) \in R^m$, $B : R^m \to L^2(\Omega)$ is a bounded linear operator.

Problem statement

The measurements are given by the output function:

$$z(x,t) = Cy(x,t), \tag{2}$$

where $C\colon L^2(\Omega \times [a,b]) \to L^2(a,b;R^m)$ is a bounded operator with dense domain, m donates the number of sensors.

How to use the regional analysis

- The considered system is said to be regional exactly controllable on ω at time b, if for every $y_b \in L^2(\omega)$, there exists a $u \in L^2([a,b], R^m)$ such that
$$p_\omega y_u(x,b) = y_b,$$
where p_ω is the restriction map from Ω to its subset ω.

- The considered system is said to be regionally exactly observable in ω, if $p_\omega y_0 \subseteq L^2(\Omega)$ can be uniquely determined by $z(x,t)$.

Methods

DPSs(distributed parameter systems): the state depends on the spatial distribution and the state space is infinite-dimensional.

Typical examples: systems described partial differential equations, integral equations and functional differential equations.

- Properties of the partial differential equations(PDEs)
- Theory of infinite-dimensional linear systems
- Semigroups and functional analysis
- Lie algebra

When we need regional analysis

- Complex systems
- Need plenty of actuators and sensors
- Difficulty in computation
- Discuss the systems which are not controllable/observable/stable/ detectable on the whole domain

Solution of system (1)-(2)

For observability problem, $Bu(t) = 0$ in (1).

Applying Laplace transform and its inverse on (1), we obtain

$$y(x,t) = G(t)y_0(x),$$

where

$$G(t) = \mathfrak{L}^{-1}\left(s^{\alpha-1}\left(\psi(1-\alpha) - \log s\right) \cdot \left(s^{\alpha}\left(\psi(1-\alpha) - \log s\right)\mathrm{I} - A\right)^{-1}\right)(t).$$

And

$$z(x,t) = CG(t)y_0(x).$$

Theoretical results

Denote $Q = CG$, $H = p_\omega Q^*$, where Q^* is the adjoint operator of Q. Then we have the following equivalent conditions.

- System (1)-(2) is regionally exactly observable in ω;
- $Im\,(H) = L^2(\omega)$;
- $Ker\,(p_\omega) + Im(Q^*) = L^2(\Omega)$;
- There is a constant $c > 0$ such that, $z \in L^2(\omega)$,

$$\|z\|_{L^2(\omega)} \leq c\,\|H^* z\|_{L^2(a,b;R^m)}\,.$$

We can also obtain some similar equivalent conditions for regional approximate observability.

Minimum number of sensors

For zone sensors, $z_i(t) = \int_{P_i} d_i(x)y(x,t)dx$, where $P_i \subseteq \Omega$ stands for the location of the i-th sensor and d_i is its corresponding spatial distribution, $i = 1, \ldots, m$.

Let r_k be the multiplicities of the k-th eigenvalue λ_k of A, $\alpha_{kj}(x)$ be the j-th eigenfunction of λ_k. Denote χ_{P_i} be the indicator function on P_i, $d^i_{kj}(x) = \langle \chi_{P_i} d_i(x), \alpha_{kj}(x) \rangle$ and define

$$D_k = \begin{bmatrix} d^1_{k1}(x) & \cdots & d^1_{kr_k}(x) \\ \vdots & \ldots & \vdots \\ d^m_{k1}(x) & \cdots & d^m_{kr_k}(x) \end{bmatrix}.$$

Then the sensors $(P_i, d_i(x))$, $i = 1, \ldots, m$ are $\omega-$strategic if and only if

$$m \geq r \triangleq \sup\{r_k\} \quad and \quad rank\, D_k = r_k, \quad for \quad k = 1, 2, \ldots.$$

Solution of system (1)

For controllability problem,

$$y(x,t) = G(t)y_0(x) + \widetilde{G}(t),$$

where

$$\widetilde{G}(t) = \mathfrak{L}^{-1}\left(\left(s^{\alpha}\left(\psi(1-\alpha) - \log s\right)\mathrm{I} - A\right)^{-1} F(s)\right)(t).$$

Here, $F(s)$ is the Laplace transform of $Bu(t)$.

Theoretical results

Define $\widetilde{H}u = y_u(x, b)$, then the following conditions are equivalent:

- System (1)-(2) is regionally exactly controllable in ω;
- $Im\left(p_\omega \widetilde{H}\right) = L^2(\omega)$;
- $Ker\,(p_\omega) + Im(\widetilde{H}) = L^2(\Omega)$;
- There is a constant $c > 0$ such that, $y \in L^2(\omega)$,

$$\|y\|_{L^2(\omega)} \leq c \left\| \widetilde{H}^* p_\omega^* y \right\|_{L^2(a,b;R^m)}.$$

We can also obtain some similar equivalent conditions for regional approximate controllability.

Minimum number of sensors

For zone sensors, $Bu = \sum_{i=1}^{m} \chi_{P_i} d_i(x) u_i(t)$. Define

$$D_k = \begin{bmatrix} d_{k1}^1(x) & \cdots & d_{kr_k}^1(x) \\ \vdots & \ldots & \vdots \\ d_{k1}^m(x) & \cdots & d_{kr_k}^m(x) \end{bmatrix}.$$

Then the actuators $(P_i, d_i(x))$, $i = 1, \ldots, m$ are $\omega-$strategic if and only if

$$m \geq r \triangleq \sup\{r_k\} \quad and \quad rank\, D_k = r_k, \quad for \quad k = 1, 2, \ldots.$$

Further Research Directions

- Optimal sensors/actuators placements
- Regional gradient observability/controllability
- Mobile sensors/actuators
- Semi-linear or nonlinear systems
- Stochastic PDEs

Main references

1 Steinbuch M. Tesla Model S battery degradation data[DB/OL]. https://steinbuch.wordpress.com/2015/01/24/tesla-model-s-battery-degradation-data/,2015-01-24/2018-06-12.

2 Sheng H, Li Y, Chen YQ. Application of numerical inverse Laplace transform algorithms in fractional calculus[J]. Journal of the Franklin Institute, 2011, 348(2):315-330.

3 https://www.mathworks.com/matlabcentral/fileexchange/3955-prony-toolbox

4 Jai AE, Pritchard AJ. Sensors and Controls in the Analysis of Distributed Systems[M]. Halsted Press, 1988.

5 Ge F, Chen YQ, Kou C. Regional Analysis of Time-Fractional Diffusion Processes[M]. Springer, 2018.

Thanks for your attention!

Questions?